Formulación, integración y uso del modelo de Molenkamp con cementación

MSc. Mauro G. Sottile
SRK Consulting (Argentina) latam.srk.com msottile@srk.com.ar

Modelo de Molenkamp extendido

Energía complementaria de deformación

$$W_{co} = \frac{1}{G[p, e_0]} \left(\frac{S\,(c_p - p)^2}{(1-m)(2-m)} + \frac{\mathbf{s}:\mathbf{s}}{4} \right)$$

Rigidez al corte

$p < 0$

$$G\,[p, e_0] = G_{ref} \frac{(c_e - e_0)^2}{1 + e_0} \left(\frac{c_p - p_c}{c_p - p} \right)^n \left(\frac{c_p - p}{c_p + p_{ref}} \right)^m$$

$p \geq 0$

$$G\,[p, e_0] = G_{ref} \frac{(c_e - e_0)^2}{1 + e_0} \left(\frac{c_p - p_c}{c_p} \right)^n \left(\frac{c_p}{c_p + p_{ref}} \right)^m$$

Modelo hiperelástico de Molenkamp extendido

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Rigidez al corte

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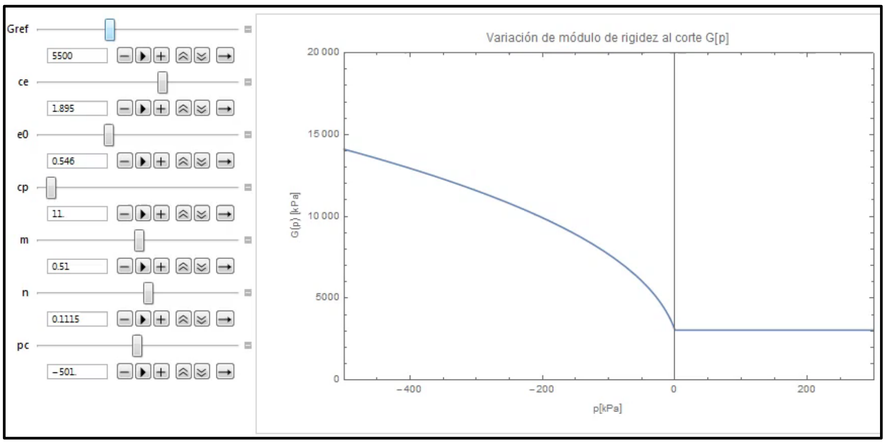

Tensor deformación y operador elástico

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Deformación

$$\begin{cases} \epsilon_v = \dfrac{\partial W_{co}}{\partial p} = \dfrac{1}{G[p, e_0]}\left(\dfrac{m\, \boldsymbol{s}:\boldsymbol{s}}{4(c_p - p)} - \dfrac{(c_p - p)\, S}{1 - m}\right) \\ \boldsymbol{\epsilon}_d = \dfrac{\partial W_{co}}{\partial \boldsymbol{s}} = \dfrac{\boldsymbol{s}}{2G[p, e_0]} \qquad S = \dfrac{3(1 - 2\nu)}{2(1 + \nu)} \end{cases}$$

Operador elástico

$$\boldsymbol{C}^e = \frac{\partial^2 W_{co}}{\partial \boldsymbol{\sigma}^2} = \frac{\partial \boldsymbol{\epsilon}}{\partial \boldsymbol{\sigma}} = \frac{1}{3}\frac{\partial \boldsymbol{\epsilon}}{\partial p} \otimes \mathbf{1} + \frac{\partial \boldsymbol{\epsilon}}{\partial \boldsymbol{s}} : \mathbf{I}^{dev} =$$

$$\frac{1}{9}\frac{\partial \epsilon_v}{\partial p}\mathbf{1} \otimes \mathbf{1} + \frac{1}{3}\frac{\partial \boldsymbol{\epsilon}_d}{\partial p} \otimes \mathbf{1} + \frac{1}{3}\mathbf{1} \otimes \frac{\partial \epsilon_v}{\partial \boldsymbol{s}} + \frac{\partial \boldsymbol{\epsilon}_d}{\partial \boldsymbol{s}}$$

Términos del operador elástico

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Modelo hiperelástico de Molenkamp extendido

$$\boldsymbol{C}^e = \frac{1}{9}\frac{\partial \epsilon_v}{\partial p}\mathbf{1}\otimes\mathbf{1} + \frac{1}{3}\frac{\partial \boldsymbol{\epsilon}_d}{\partial p}\otimes\mathbf{1} + \frac{1}{3}\mathbf{1}\otimes\frac{\partial \epsilon_v}{\partial \boldsymbol{s}} + \frac{\partial \boldsymbol{\epsilon}_d}{\partial \boldsymbol{s}}$$

Compresión	Tracción
$\frac{\partial \epsilon_v}{\partial p} = \frac{1}{G[p,e_0]}\left(S + \frac{m(1-m)\boldsymbol{s}:\boldsymbol{s}}{4(c_p - p)^2}\right)$	$\frac{\partial \epsilon_v}{\partial p} = \frac{S}{(1-m)G[p,e_0]}$
$\frac{\partial \boldsymbol{\epsilon}_d}{\partial p} = \frac{\partial \epsilon_v}{\partial \boldsymbol{s}} = \frac{1}{2G[p,e_0]}\frac{m\boldsymbol{s}}{(c_p - p)}$	$\frac{\partial \epsilon_v}{\partial \boldsymbol{s}} = \frac{m\boldsymbol{s}}{2\,c_p\,G[p,e_0]}$
$\frac{\partial \boldsymbol{\epsilon}_d}{\partial \boldsymbol{s}} = \frac{\mathbf{I}^{dev}}{2G[p,e_0]}$	$\frac{\partial \boldsymbol{\epsilon}_d}{\partial \boldsymbol{s}} = \frac{\mathbf{I}^{dev}}{2G[p,e_0]}$ $\quad \frac{\partial \boldsymbol{\epsilon}_d}{\partial p} = \mathbf{0}$

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Algoritmo de actualización

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Modelo hiperelástico de Molenkamp extendido

- Se recibe $\boldsymbol{\sigma}_n$ y $\Delta\boldsymbol{\epsilon}$
- Se calcula $\boldsymbol{\epsilon}^n$ y $\boldsymbol{\epsilon}^{n+1} = \boldsymbol{\epsilon}^n + \Delta\boldsymbol{\epsilon}$
- En ϵ_v se reemplaza $s^{n+1} = 2G[p,e_0]\epsilon_d^{n+1}$

$$\epsilon_v[p] = \frac{1}{G[p,e_0]}\left(\frac{m\,G^2[p,e_0]}{4(c_p - p)}\boldsymbol{\epsilon}_d:\boldsymbol{\epsilon}_d - \frac{(c_p - p)\,S}{1-m}\right)$$

- Se calcula p^{n+1} minimizando $F[p] = \epsilon_v\,[p] - \epsilon_v^{n+1}$

$$F^{n+1} = \frac{1}{G[p,e_0]}\left(\frac{m\,G^2[p,e_0]}{4(c_p - p)}\boldsymbol{\epsilon}_d:\boldsymbol{\epsilon}_d - \frac{(c_p - p)\,S}{1-m}\right) - \epsilon_v^{n+1} = 0$$

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Algoritmo de actualización

```
(* Converged state *)
p0 = CalcP [ Sig0 ] ;
s0 = CalcDev [ Sig0 ] ;
pc = Min [ 0 , pc , p0 ] ;
(* Converged total strain *)
If [ p0 < 0 ,
    Ge = G1 * ( cp - p0 ) ^ mn ;
    Epsv0 = ( mn * DContr [ s0 , s0 ] / ( 4 * ( cp - p0 ) ) - ( cp - p0 ) * S / ( 1 - mn ) ) / Ge ;
    Epsd0 =  s0 / ( 2 * Ge ) ;
,
    Ge = G1 * cp ^ mn;
    Epsv0 = ( mn * DContr [ s0 , s0 ] / ( 4 * cp ) - ( cp - p0 ) * S / ( 1 - mn ) ) / Ge ;
    Epsd0 =  s0 / ( 2 * Ge ) ;
] ;
(* Strain increment *)
dEpsv = CalcI1 [ dEps ] ;
dEpsd = CalcDev [ dEps ] ;
(* Updated total strain *)
Epsv = Epsv0 + dEpsv ;
Epsd = Epsd0 + dEpsd ;
DCEpsd = DContr[ Epsd , Epsd ] ;
Ge = G1 * cp ^ mn ;
```

Modelo hiperelástico de Molenkamp extendido

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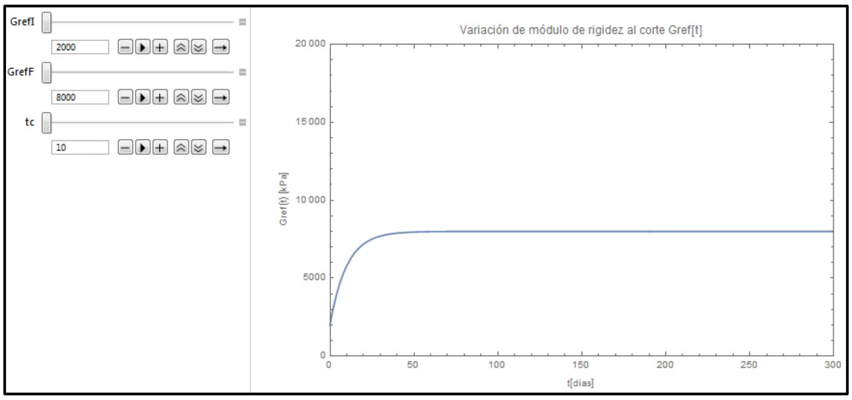
Cementación
(cambio de la rigidez con la edad)

$$G_{ref}[t] = (G_{ini} - G_{fin})e^{-\frac{t}{t_{fin}}} + G_{fin}$$



Modelo hiperelástico de Molenkamp extendido

Verificación numérica

Hiperelasticidad

Modelo de Molenkamp

```
(* VERIFICACION DEL MODELO PARA DIFERENTES PASOS DE CARGA *)
(* Definicion de Parametros *)
Clear [ Pars , StVar , Incr , σ , Δϵ , e0 ]
EModel = 7 ;
Pars = { 10000 , 2.17 , 1.01, 0.5 , 0.2 , 100 } ; (* { Gref , ce , cp , m , ν , pref } *)


(* Definicion del estado inicial de tensiones *)
σ = 100 * { -100 * RandomReal [ ] , -100 * RandomReal [ ] , -100 * RandomReal [ ] , RandomReal [ ] - 0.5 , RandomReal [ ] - 0.5 , RandomReal [ ] - 0.5 } ;
(* Definicion del incremento de deformaciones *)
Δϵ = 0.01 * { RandomReal [ ] - 0.6 , RandomReal [ ] - 0.6 , RandomReal [ ] - 0.6 , RandomReal [ ] - 0.5 , RandomReal [ ] - 0.5 , RandomReal [ ] - 0.5 } ;

e0 = 0.51 ;
StVar = { σ , e0 } ;
Incr = { Δϵ } ;

(* 1 Paso de Carga *)
{ σnew , De } = UpdateHyperEl [ EModel , Pars , StVar , Incr ] ;
Print [ "1 Step:" , σnew ] ;
```

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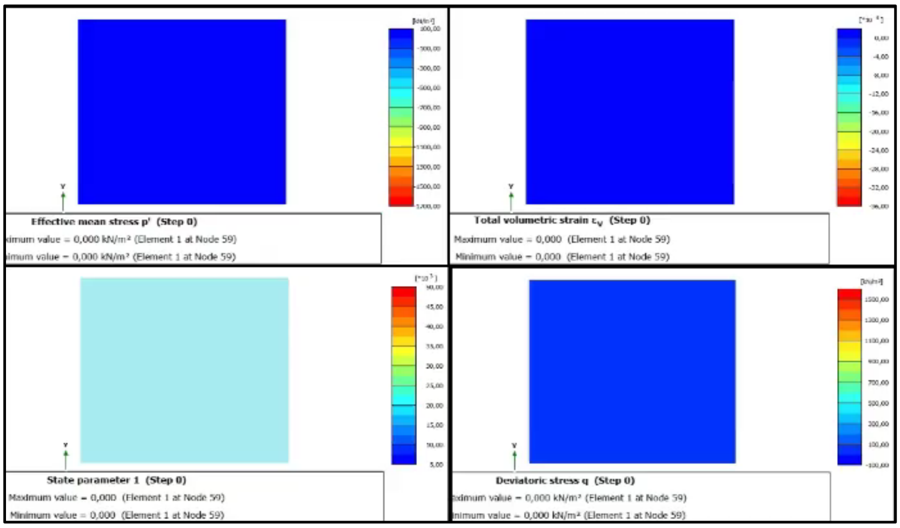
Modelo hiperelástico de Molenkamp extendido

Aplicación: base flexible en Plaxis 2D



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