Tipos de elemento
Ejercicio elemento cuatro nodos

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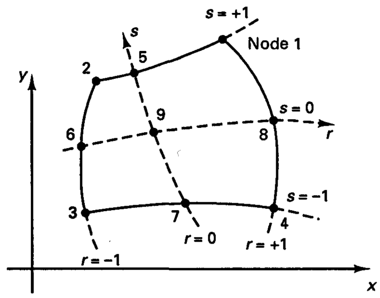
Elemento 2D de 4 a 9 nodos

Para aumentar la precisión se pueden agregar más elementos o usar elementos con mas nodos

Cuadrado

- 4 nodos: interpolación bi-lineal



(a) 4 to 9 variable-number-nodes two-dimensional element

Include only if node i is defined

		$i = 5$	$i = 6$	$i = 7$	$i = 8$	$i = 9$
$h_1 =$	$\frac{1}{4}(1+r)(1+s)$	$-\frac{1}{2}h_5$			$-\frac{1}{2}h_8$	$-\frac{1}{4}h_9$
$h_2 =$	$\frac{1}{4}(1-r)(1+s)$	$-\frac{1}{2}h_5$	$-\frac{1}{2}h_6$			$-\frac{1}{4}h_9$
$h_3 =$	$\frac{1}{4}(1-r)(1-s)$		$-\frac{1}{2}h_6$	$-\frac{1}{2}h_7$		$-\frac{1}{4}h_9$
$h_4 =$	$\frac{1}{4}(1+r)(1-s)$			$-\frac{1}{2}h_7$	$-\frac{1}{2}h_8$	$-\frac{1}{4}h_9$
$h_5 =$	$\frac{1}{2}(1-r^2)(1+s)$					$-\frac{1}{2}h_9$
$h_6 =$	$\frac{1}{2}(1-s^2)(1-r)$					$-\frac{1}{2}h_9$
$h_7 =$	$\frac{1}{2}(1-r^2)(1-s)$					$-\frac{1}{2}h_9$
$h_8 =$	$\frac{1}{2}(1-s^2)(1+r)$					$-\frac{1}{2}h_9$
$h_9 =$	$(1-r^2)(1-s^2)$					

(Bathe 1996)

Ejercicio elemento 4 nodos

Ejemplo (mecánica continuo)

$$\frac{\partial u_1}{\partial x_1} = \frac{\partial[1.5+0.1xy]}{\partial x} = 0.1y$$

$$\frac{\partial u_1}{\partial x_2} = \frac{\partial[1.5+0.1xy]}{\partial y} = 0.1x$$

$$\frac{\partial u_2}{\partial x_1} = \frac{\partial[0.5+0.1xy]}{\partial x} = 0.1y$$

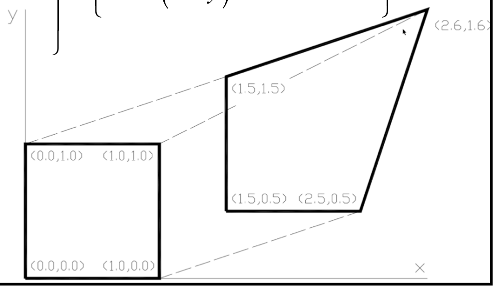
$$\frac{\partial u_2}{\partial x_2} = \frac{\partial[0.5+0.1xy]}{\partial y} = 0.1x$$

$$\underline{u}[x,y] = \underline{x}^{end} - \underline{x}^{ini} = \begin{Bmatrix} 1.5+0.1xy \\ 0.5+0.1xy \end{Bmatrix}$$

$$\underline{u}[0.5,0.4] = \begin{Bmatrix} 1.5+0.1x0.5x0.4 \\ 0.5+0.1x0.5x0.4 \end{Bmatrix} = \begin{Bmatrix} 1.52 \\ 0.52 \end{Bmatrix}$$

$$\underline{\underline{\varepsilon}}[x,y] = \begin{Bmatrix} \frac{\partial u_1}{\partial x_1} & \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right) \\ \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right) & \frac{\partial u_2}{\partial x_2} \end{Bmatrix} = \begin{Bmatrix} 0.1y & 0.05(x+y) \\ 0.05(x+y) & 0.1x \end{Bmatrix}$$

$$\underline{\underline{\varepsilon}}[0.5,0.4] = \begin{Bmatrix} 0.04 & 0.045 \\ 0.045 & 0.05 \end{Bmatrix}$$



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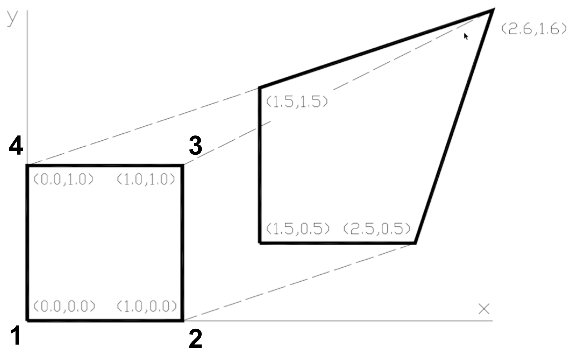
Ejercicio elemento 4 nodos

Ejemplo 2D: desplazamientos FEM



$$u_e = \mathbf{H}\cdot U_e = \mathbf{H}\cdot\begin{Bmatrix} U_1 \\ V_1 \\ U_2 \\ V_2 \\ U_3 \\ V_3 \\ U_4 \\ V_4 \end{Bmatrix} = \mathbf{H}\cdot\begin{Bmatrix} 1.5 \\ 0.5 \\ 1.5 \\ 0.5 \\ 1.6 \\ 0.6 \\ 1.5 \\ 0.5 \end{Bmatrix}$$

u_e = f[U_e] = H x U_e

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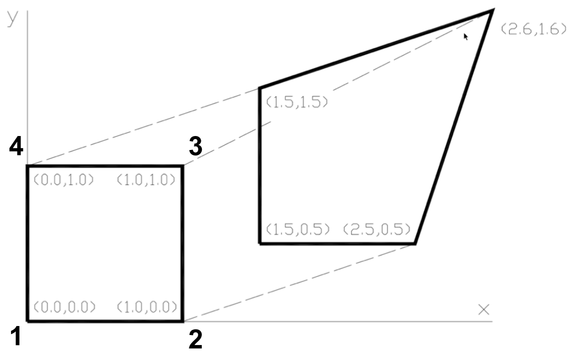
Ejemplo 2D: desplazamientos FEM



$$\mathbf{H}=\begin{bmatrix}(1-x)(1-y) & 0 & x(1-y) & 0 & xy & 0 & (1-x)y & 0\\ 0 & (1-x)(1-y) & 0 & x(1-y) & 0 & xy & 0 & (1-x)y\end{bmatrix}$$

Ejemplo 2D: desplazamientos FEM

$$\mathbf{H}=\begin{bmatrix}(1-x)(1-y) & 0 & x(1-y) & 0 & xy & 0 & (1-x)y & 0\\ 0 & (1-x)(1-y) & 0 & x(1-y) & 0 & xy & 0 & (1-x)y\end{bmatrix}$$

$$u_e=\mathbf{H}\cdot U_e=\begin{bmatrix}0.3 & 0 & 0.3 & 0 & 0.2 & 0 & 0.2 & 0\\ 0 & 0.3 & 0 & 0.3 & 0 & 0.2 & 0 & 0.2\end{bmatrix}\begin{Bmatrix}1.5\\0.5\\1.5\\0.5\\1.6\\0.6\\1.5\\0.5\end{Bmatrix}=\begin{Bmatrix}1.52\\0.52\end{Bmatrix}$$

u_e = f[U_e] = H x U_e

Ejemplo 2D: deformaciones FEM

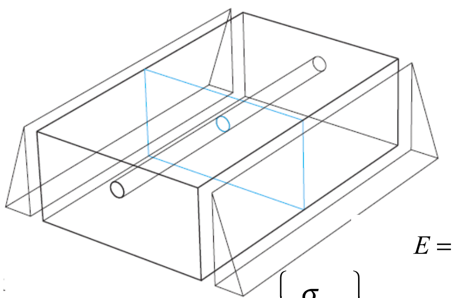
$$\mathbf{B} = \frac{\partial \mathbf{H}}{\partial \underline{x}} = \begin{bmatrix} -(1-y) & 0 & 1-y & 0 & y & 0 & -y & 0 \\ 0 & -(1-x) & 0 & -x & 0 & x & 0 & 1-x \\ -(1-x)/2 & -(1-y)/2 & -x/2 & (1-y)/2 & x/2 & y/2 & (1-x)/2 & -y/2 \end{bmatrix}$$

$$\varepsilon = \mathbf{B}\cdot\mathbf{U}_e = \begin{bmatrix} -0.6 & 0 & 0.6 & 0 & 0.4 & 0 & -0.4 & 0 \\ 0 & -0.5 & 0 & -0.5 & 0 & 0.5 & 0 & 0.5 \\ -0.25 & -0.3 & -0.25 & 0.3 & 0.25 & 0.2 & 0.25 & -0.2 \end{bmatrix} \begin{Bmatrix} 1.5 \\ 0.5 \\ 1.5 \\ 0.5 \\ 1.6 \\ 0.6 \\ 1.5 \\ 0.5 \end{Bmatrix} = \begin{Bmatrix} 0.04 \\ 0.05 \\ 0.045 \end{Bmatrix}$$

u_e = H x U_e

ε = B x U_e

Ejemplo 2D: tensiones (elasticidad)



$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix}$$

$$E = 1000\,MPa \quad \nu = 0.20$$

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \frac{1000\,MPa}{1-0.2^2} \begin{bmatrix} 1 & 0.2 & 0 \\ 0.2 & 1 & 0 \\ 0 & 0 & 0.8 \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{xy} \end{Bmatrix}$$

u_e = f[U_e]

ε = f[u_e]

σ = f[ε]

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix}_{[0.5,0.4]} = \frac{1000\,MPa}{1-0.2^2} \begin{bmatrix} 1 & 0.2 & 0 \\ 0.2 & 1 & 0 \\ 0 & 0 & 0.8 \end{bmatrix} \begin{Bmatrix} 0.04 \\ 0.05 \\ 0.045 \end{Bmatrix} = \begin{Bmatrix} 52 \\ 60 \\ 38 \end{Bmatrix} \text{MPa}$$